ESC194

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1 Previous Lectures:

Linear first order:

$$y' + p(x)y = q(x)$$

$$\rightarrow y = e^{-H(x)} \left[\int e^{H(x)} q(x) dx + c \right]$$

when H(x) = $\int p(x) dx \quad e^{H(x)} = \int p(x) dx$

Example:

$$\frac{dP}{dt} = kP(1 - \frac{P}{M})$$

let $z = \frac{1}{P}$

$$\rightarrow P = \frac{1}{Z} \rightarrow \frac{dP}{dt} = \frac{-1}{Z^2} \frac{dZ}{dt}$$

$$-\frac{1}{z^2} \frac{dz}{dt} = \frac{k}{z}(1 - \frac{1}{zm})$$

$$\frac{d}{z}dt = -kz(1 - \frac{1}{zm}) \rightarrow \frac{dz}{dt} + kz = \frac{k}{M}$$

$$H(t) = \int k dt = kt \rightarrow IF = e^{kt}$$

$$\therefore Z = e^{kt} \left[\int e^{kt} \frac{k}{M} dt + c \right]$$

$$= e^{-kt}\left[\frac{e^{kt}}{M} + c\right]$$

$$= \frac{1}{M} + Ce^{-kt}$$

Circuit Example Again, New Diagram:

Let's ignore the diagram since this isn't fucking electronics class, here are the governing equations:

$$P = \frac{1}{Z} = \frac{M}{1 + MCe^{-kt}}$$

$$E(t) = \frac{Q}{C} + IR$$

$$I = \frac{dQ}{dt}$$

$$\therefore E(t) = \frac{Q}{C} + \frac{RdQ}{dt}$$

$$Q' + \frac{1}{Rc}Q = \frac{E(t)}{R}$$

Now doing the maths:

$$p(t) = \frac{1}{RC}$$

$$q(t) = \frac{E(t)}{r}$$

$$H(t) = \int \frac{1}{RC}dt = \frac{t}{RC} \to IF = e^{\frac{t}{RC}}$$

$$\therefore \frac{d}{dt}(e^{\frac{t}{RC}} \cdot Q) = e^{\frac{t}{RC}}\frac{E(t)}{R}$$

$$\to Q = e^{\frac{t}{RC}}\left[\int e^{\frac{t}{RC}}\frac{E(t)}{R}dt + A\right]$$

Case 1:

$$E(t) = E = const$$

$$\therefore Q = e^{-\frac{t}{RC}}\left[\frac{E}{R} \cdot RCe^{\frac{t}{RC}}\right] = EC + Ae^{-\frac{t}{RC}}$$

We get an exponential rise up of Q asymptotically to the line Q = EC compared to time.

Case 2:

$$E(t) = \alpha sin(\beta t)$$

$$Q = e^{-\frac{t}{RC}} \int e^{\frac{t}{RC}} \frac{\alpha sin(\beta t)}{R} + Ae^{-\frac{t}{RC}}$$

[this line was fucking unreadable]

$$\therefore Q = \frac{\alpha/R}{(\frac{t}{RC})^2 + \beta^2} \left(\frac{t}{RC} sin\beta t - \beta cos(\beta t) \right) + Ae^{-t/RC}$$

Bro what is going on

Example:

$$y' - 4y = 3e^x y^{1/2}$$

$$y^{-1/2} y' - 4y^{1/2} = 3e^x$$

substitution:
let $u = y^{1/2} \to u' = \frac{1}{2} y^{-1/2} y'$

$$\therefore 2y' - 4u = 3e^x \to u' 2u = \frac{3}{2} e^x$$

$$y' + p(x)y = q(x)y^r; \quad r \neq 0, 1$$

$$u = y^{1-r} \to u' + (1-r)p(x)u = (1-r)q(x)$$

This is considered a **Bernoulli Equation**

$$H(x) = \int -2x dx \to IF = e^{-2x}$$

$$\to u = e^{2x} \left[\int \frac{3}{2} e^x - e^{-2x} dx + c \right] = -\frac{3}{2} + Ce^{2x}$$

$$y = u^2 \to y = \left(-\frac{3}{2} e^x + Ce^{2x} \right)^2$$

2 Complex Numbers

$$x^2 = 2$$
$$\rightarrow x = \sqrt{2}$$

Argument for including irrational numbers: we want as many numbers as possible to allow for use in our number system.

Now consider:

$$x^2 = -1$$

$$i^2 = -1 \text{ or } i = \sqrt{-1}$$

Complex numbers are written in the form:

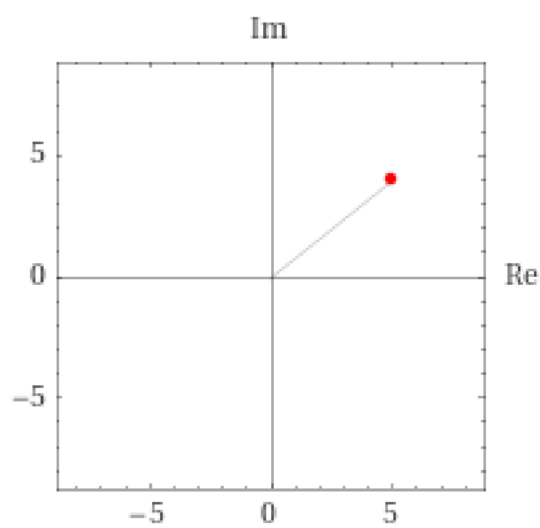
$$z = a + ib; \quad a, b \text{ are real numbers}$$

a is the real part, and b is the imaginary part.

$$a + ib \rightarrow (a, b)$$

We can plot imaginary numbers on a graph!:

For example, plotting 4i + 5 we get:



These are called argan diagrams.

$$\text{Complex plane: } C = \{a + ib : a, b \text{ real numbers}\}$$

We can also consider imaginary numbers in polar coordinate form, consider a point z. The distance from the origin is known as the *modulus* of the complex number, denoted as $|z|$, and given by:

$$|a + ib| = \sqrt{a^2 + b^2}$$

We can also consider the angle θ between the x axis and the line drawn out from the origin to our point z. This angle is called the *argument*, denoted as $\theta = arg(z)$. You could find this angle either using inverse trig functions and a calculator, or perhaps using special triangles

$$\arg(z) = \theta \rightarrow \arg(z) = \theta + 2k\pi$$

where k is any integer

Trig: Getting a and b from z

$$|z| \cdot cos(arg(z)) = a$$

$$|z| \cdot sin(arg(z)) = b$$

Examples using that:

$$|2| = 2 \rightarrow \arg(2) = 0$$

$$|1 + i| = \sqrt{2} \rightarrow \arg(1 + i) = \frac{\pi}{4}$$

$$|i| = 1 \rightarrow \arg(i) = \frac{\pi}{2}$$

$$|-\sqrt{3} + 2| = 2 \rightarrow \arg(-\sqrt{3} + i) = \frac{5\pi}{6}$$

R?:

$$r = |z| = \sqrt{a^2 + b^2}$$

Polar representation:

$$z = rcos\theta + irsin\theta$$

Definition: Complex conjugate

$$z = a = ib \rightarrow \bar{z} = a - ib$$

$$z = 2 - 3i \rightarrow \bar{z} = 2 + 3i$$

$$z = 2 \rightarrow \bar{z} = 2$$

$$z = -3i \rightarrow \bar{z} = +3i$$

3 Complex Arithmetic:

$$z = a + ib \qquad w = c + id$$

$$z + w = (a + c) + i(b + d)$$

$$z - w = (a - c) + i(b - d)$$

$$z + w = w + z \qquad \text{(commutative)}$$

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3) \qquad \text{(associative)}$$

Triangle Inequality:

$$|z_1 \pm z_2| \leq |z_1| + |z_2|$$

$$(z \bar{+} w) = \bar{z} + \bar{w}$$